Thermal and electrical influences from bulk plasma in cathode heating modeling

This content has been downloaded from IOPscience. Please scroll down to see the full text.
2017 Plasma Sources Sci. Technol. 26 025002
(http://iopscience.iop.org/0963-0252/26/2/025002)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 202.38.91.74
This content was downloaded on 26/12/2016 at 02:41

Please note that terms and conditions apply.

You may also be interested in:

Diffuse and spot mode of cathode arc attachments in an atmospheric magnetically rotating argon arc
Tang Chen, Cheng Wang, Meng-Ran Liao et al.

Heating of refractory cathodes
M S Benilov and M D Cunha

Novel non-equilibrium modelling of a DC electric arc in argon
M Baeva, M S Benilov, N A Almeida et al.

Understanding and modelling plasma–electrode interaction in high-pressure arc discharges
M S Benilov

Modelling of a high-pressure arc plasma
He-Ping Li and M S Benilov

Account of near-cathode sheath in numerical models of high-pressure arc discharges
M S Benilov, N A Almeida, M Baeva et al.

Unified modelling of near-cathode plasma layers in high-pressure arc discharges
N A Almeida, M S Benilov and G V Naidis

A model of the cathode region of atmospheric pressure arcs
M S Benilov and A Marotta

Sheath and arc-column voltages in high-pressure arc discharges
M S Benilov, L G Benilova, He-Ping Li et al.
Thermal and electrical influences from bulk plasma in cathode heating modeling

Tang Chen¹, Cheng Wang¹, Xiao-Ning Zhang², Hao Zhang¹ and Wei-Dong Xia¹

¹ Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei 230027, People’s Republic of China
² Laboratory for Space Environment and Physical Sciences, Harbin Institute of Technology, Harbin 150001, People’s Republic of China

E-mail: xiawd@ustc.edu.cn

Received 28 January 2016, revised 2 October 2016
Accepted for publication 9 November 2016
Published 23 December 2016

Abstract
In this paper, a numerical calculation is performed for the purpose of estimating the thermal and electrical influences from bulk plasma in cathode heating modeling, in other words researching the necessity of a coupling bulk plasma in near-cathode layer modeling. The proposed model applied in the present work is an improved one from previous work.

In this model, the near-cathode region is divided into two parts: the sheath and the ionization layer. The Schottky effect at the cathode surface is considered based on the analytic solution of a 1D sheath model. It is noted that the arc column is calculated simultaneously in the near-cathode region and the cathode bulk. An application is presented for an atmospheric free burning argon arc with arc currents of 50 A–600 A.

The modeling results show three interesting points: (1) at the cathode surface, energy transport due to heat conduction of heavy particles and electrons is comparable to total heating flux, no matter whether the arc discharge is performed in a high (400 A) or low current (50 A) situation; (2) the electrical influence from bulk plasma on the cathode heating modeling becomes obvious in a high current situation (>400 A) for the spot mode; (3) the near-cathode layer voltage drop ($U_{tot}$) is larger in the diffuse mode than in the spot mode for the same current, which is just the opposite to that for decoupled modeling.

Keywords: arc-cathode interaction, spot mode, diffuse mode, arc discharge

(Some figures may appear in colour only in the online journal)
**1. Introduction**

Unified modeling for arc plasmas and a thermionic cathode at high-pressure has long been the subject of extensive studies, thanks to the increasing power of computers. The experimental difficulties of using arc plasmas in the near-electrode region is obvious, due to the sharp change in plasma state over a very small distance [1–6]. Successful near-anode region modeling and experiments have been carried out by Tanaka, Ushio and coworkers [7–11], and Dinulescu [12]. On the other hand it has been found [13] that the near-cathode region involves highly non-linear multi-field coupling, which makes the complete simulation of arc–cathode interactions quite a time-consuming job. Thus, the near-cathode layer is not generally taken into account in many models, in which case one may just impose an arbitrary current density profile at the cathode surface [14]. Nevertheless, it is essential to develop a unified arc–cathode model for the following reasons. Experimental results have shown that there is evident arc–cathode interaction, which affects the configurations of the arc cathodic root as well as the arc column [15]. This is very useful for researching practical applications in controlling the arc column configurations and cathodic root configurations [16, 17], such as a magnetically dispersed arc plasma [18, 19], diffuse cathodic root [20], etc. To research arc discharge phenomena using a thermionic cathode, a model including the electrodes, the near-electrode layer and the arc column was successfully attempted by Lowke, Sansonnes and coworkers [21–24], Benilov, Li and coworkers [25, 26], Murphy, Schnick and coworkers [27–31].

Different models of plasma–cathode interaction in high-pressure arc discharges are systematized in [32]. To the authors’ knowledge, the near-cathode layer can be treated as a region either dependent or independent of bulk plasma. In Lowke’s work, the 2D non-equilibrium layer coupled with plasma was assumed to be in the local thermodynamic equilibrium state, except that effective thermal and electrical conductivities are calculated for the first mesh cells adjacent to the cathode surface [22]. The space charge layer was neglected and a 2D ionization layer was applied. In some cases of simulation [21, 22], the cathode surface temperatures seem to be in good agreement with the experimental data. However, the no-sheath approach physically conflicts with experimental facts.
[25]: in low-current arcs \( I \leq 10 \) A, the values of cathode fall are in the range of 10–50 V; in high-current arcs \( I = 50–500 \) A, the cathode fall is around 4–10 V. Thus for a further study on the interaction between the arc and the cathode, a sheath-accounted approach is required. Simulations from [33] developed a novel non-equilibrium model of a DC electric arc, with space-charge sheaths adjacent to the electrodes. Represented by the theory of Benilov [34], those models involve both the sheath and ionization layer in a 1D description [25, 26, 34–43]. The modeling, which decouples the cathode from the plasma column, was first suggested by Bade and Yos [44] and Neumann [45], and then developed by Benilov [34, 38–40] and other researchers [3, 4, 35–37, 44–47]. These above-mentioned models included the sheath but approximately assume that the ion current fraction in the total current does not vary much under different arc column conditions. In addition, the energy balance of the near-cathode layer is assumed to be independent of the arc column. In high-pressure low-current situations (e.g. high intensity discharge (HID) lamp discharge), this kind of theory is proved successfully compared to the experimental measurements [3, 4, 39, 46], and hence is widely accepted. Li, Benilov and coworkers [25, 26] tried to introduce the sheath-accounted model [39] to the high-current arc situation and prove the primary importance of near-cathode sheaths.

The basic assumptions of the decoupling method and the coupling method deviate greatly from each other. Until now, there has been no adequate strong proof for neglecting the influence of bulk plasma on the cathode arc attachment for high-pressure high-current arc discharges, e.g. cutting, welding or waste treatment. Plasma flow due to a strong magnetic field at the cathode root (the Mæcker effect) could be important at high arc currents, for example, the arc root could shrink towards its tip under an external magnetic field [16]. The cathode arc attachment was shown to be greatly influenced by the plasma flowing in rotating arc simulation [15]. Further work is required to clarify whether the decoupling method is appropriate for high-current arc modeling.

Therefore in this paper, for the purpose of examining the thermal and electrical influence of bulk plasma on cathode heating, we present a two temperate arc column model coupled with 1D near-cathode layer modeling, which is developed from previous work [48]. The modeling and calculation method in this paper is basically same as a previous work [48]; however, there are several small modifications (such as the expression of electron enthalpy, and, most importantly, the electrical influence from bulk plasma on the near-cathode layer is not included in the calculation of [48]. The conduction heating flux is included in the electron energy balance equation of the ionization layer. To achieve a more accurate result, a thin thermal perturbation layer is taken into account, which is in both a chemical and thermodynamic non-equilibrium state. The thermal perturbation layer is defined as a layer in which electron thermal conduction and heating of the electrons by an ambipolar electric field are comparable to electron energy exchange in collisions with heavy particles [34, 49]. The arc model considers a tungsten rod cathode and a flat anode.

In the second part of this paper, detailed descriptions of the theory are presented. We give the descriptions of the case geometry and then a summary of the basic assumptions, the equations and boundary conditions.

In the third part, the general results of arc discharge, as current density distributions on the cathode tip, sheath voltage drop and the energy fluxes are presented.

In the last part, the summary of the main conclusions concerning the arc–cathode interaction will be presented.

2. Description of the numerical model

2.1. Concept of the coupling method

The concept of the modeling structure is illustrated in figure 1. \( U_i \) and \( U_d \) represent the voltage drop in the ionization layer/sheath layer; \( U_{arc} \) represents the whole arc discharge voltage; \( U_{U_4} \) represents bulk plasma voltage drop. In our previous work, the electrical influence from the bulk plasma on the near-cathode layer was not included in the calculation [48] (this means \( U_i + U_d \) is uniform in the cathode arc attachment), however in our present work \( U_{arc} = U_i + U_d + U_{id} \) is assumed to be uniform, giving a certain total current. The idea that Ohm’s law should be included in the near-cathode layer modeling, which leads to cathode voltage drop variation along the cathode surface, is not new. This idea has been developed in a series of works by Nemchinsky [3, 46, 50–53]. The basic assumptions for each layer introduced are summarized in table 1. The plasma region in front of the hot cathode is divided into three subzones [49, 54]: a space-charge sheath, an ionization layer, and a layer of thermal perturbation (or thermal relaxation). The thermal perturbation layer is established for the purpose of introducing the arc column’s influence on the near-cathode layer. This layer is assumed to be a thin layer with length 200 \( \mu m \), between the near-cathode layer and the arc column. Alternatively, one can take this layer as a part of the arc column. The basic assumptions taken for the thermal perturbation layer are summarized as follows: (a) the layer is in the thermal non-equilibrium and chemical non-equilibrium state; (b) the layer is a flow-affected 2D region; (c) the layer is supposed to be in a quasi-neutral state, which leads to an ambipolar diffusion approximation for the calculation of electron number density.

As table 1 shows, these zones may be characterized by the following length scales, respectively: the length of electron energy relaxation \( \lambda_{ee} \); the ionization length \( \lambda_i \); the Debye length \( \lambda_D \) [34]. The length of electron energy relaxation \( \lambda_{ee} \) is defined as a length scale on which electron thermal conduction and heating of the electrons by the ambipolar electric field are comparable to electron energy exchange in collisions with heavy particles [34]; the ionization length \( \lambda_i \) represents a distance to the cathode surface in which deviations from ionization equilibrium are validated.

The typical modeling of plasma boundary layer of high intensity discharge (HID) cathodes [47], maintains that the cathodic plasma boundary layer in front of a thermionically emitting cathode is independent of the bulk plasma. Following this idea the current transfer and power flux density across the boundary layer is locally 1D and governed by local values of the cathode temperature \( T_e \) and of the voltage \( U_{id} \) applied to the ionization layer and the sheath [34]:
\[ q_{\text{tot}} = \dot{j}_{\text{em}} \left( U_{\text{tot}} - A_{\text{ef}} - (2.5 + 0.7\delta) \frac{k_B T_e}{e} \right) \]
\[ - q_{\text{rad}} - k_e \nabla \cdot \bar{n} - k_h \nabla \cdot \bar{n} \]
(3)

and the electron energy balance equation in ionization layer is rewritten as:
\[ \dot{j}_{\text{em}} \frac{e}{2} (2k_B T_e + eU_e - \Delta A) + \rho - \dot{j}_{\text{rad}} \left( 2k_B T_{\text{ea}} + eU_e - \Delta A \right) + \rho \frac{e}{2} \]
(4)

where \( \dot{j}_{\text{em}} \) is the work of the electric field over electrons, \( W_c \) is the work of the electric field over plasma, \( W_c = \dot{j}_{\text{em}} U_i - 0.5j_{\text{bd}} U_i \), following [38, 39]. \( \Delta A = \frac{e k_B}{\sqrt{4 \pi n_0}} \) is the Schottky correction to the work function. Note that [38, 39] did not include \( \Delta A \) in the electron energy balance while other simulations such as [47] did. Though the results with and without the \( \Delta A \) term in equation (4) are virtually the same, we suggest in future that the related simulation does not include the \( \Delta A \) term, because the equation with the \( \Delta A \) term implies that there is a cathode voltage drop with a zero electric field at the cathode surface, which is not physically acceptable.

Therefore the total energy flux transported to electrons \( q_{\text{fe}} \) at the near-cathode layer boundary is evaluated as (using equation (4)):
\[ q_{\text{fe}} = \dot{j}_{\text{em}} (2k_B T_{\text{ea}} + eU_e - \Delta A) + W_c - \dot{j}_{\text{rad}} (2k_B T_e + eU_e - \Delta A) - \rho \frac{e}{2} \]
(5)

The detailed expressions for each component of current density are given in appendix A.

2.2. Conservation equations

2.2.1. Arc column and thermal perturbation region. In this work, the arc column is of the axisymmetrical two-temperature type. Therefore the energy equations are general in both the arc column and the thermal perturbation region.

Continuity equation:
\[ \nabla \cdot (\rho \dot{V}) = 0. \]
(6)

Momentum conservation equation:
\[ \rho (\dot{V} \cdot \nabla) \dot{V} = \nabla \cdot (-p I + \tau) + \dot{j} \times \vec{B}. \]
(7)

Here \( \dot{V} \), \( \rho \) and \( p \) are the mass-averaged velocity, density and pressure of the gas, respectively, \( I \) is the identity matrix, \( \tau \) denotes the viscous stress tensor for Newtonian fluid, and \( \dot{j} \) and \( \vec{B} \) are the current density and magnetic field.

The heavy particle energy conservation equation is:
The electron energy conservation equations [26]:

\[
\nabla \cdot \left[ -(k_a + k_i + k_e) \nabla T_e + \frac{5}{2} (n_a + n_i) k_b T_h \nabla V \right] = \dot{E}_{el} + S_e. \tag{8}
\]

The electron energy conservation equations [26]:

\[
\nabla \cdot \left[ -k_e \nabla T_e + \left( \frac{5}{2} + 0.7 \delta \right) k_b T_h \nabla e - n_i T_e \nabla V \right] = -\dot{E}_{el} + S_e \tag{9}
\]

\[
S_e = \vec{j} \cdot \vec{E} - U_t \tag{10}
\]

\[
\dot{E}_{el} = \frac{2 m_e}{m_h} (\pi_{ea} + \pi_{ei}) n_e \frac{3}{2} k_h (T_e - T_h) \tag{11}
\]

where \(T_h, T_e, k_a, k_i, k_e, \) and \(k_e\) represent the heavy particle temperature, electron temperature, thermal conductivity of atoms, thermal conductivity of ions, reactive thermal conductivity, and thermal conductivity of electrons, respectively. The thermal conductivity for the heavy particle energy equation is \(k = k_a + k_i + k_e, \) \(E_{el} \) is the energy transferred in elastic collisions between electrons and heavy particles, which can be expressed as equation (11) shows. In particular, the electron flux \(\pi_e\) is set to \(n_e \nabla V - \frac{1}{2} \) in the arc column region. \(S_e, U_t, \pi_{ea}, \) and \(\pi_{ei}\) represent the electron energy conservation source term, net volumetric rate of radiation loss, average collision frequency between electrons and atoms, and average collision frequency between electrons and ions.

\[
\pi_{ea} = C_e n_a Q_{ea} \tag{12}
\]

where \(C_e, Q_{ea}, Q_{ei}, n_a, n_i, k_b, \) and \(m_e\) represent the average thermal speed of electrons, collision cross section between electrons and atoms, collision cross section between electrons and ions, ion number density, atom number density, Boltzmann constant and the electron mass, respectively.

The electric potential and magnetic vector potential equations are:

\[
\nabla \cdot (-\sigma \nabla \varphi + eD_e \nabla n_e) = 0 \tag{15}
\]

\[
\nabla \times \vec{A} = -\xi_0 \vec{j} \tag{16}
\]

\[
\vec{B} = \nabla \times \vec{A}, \vec{j} = -\sigma \nabla \varphi + eD_e \nabla n_e \tag{17}
\]

where \(\sigma\) represents the electric conductivity, and \(\xi_0\) is the permeability of vacuum. The electric potential and magnetic vector potential equations are solved in the arc column region, ionization layer and cathode region. The second right-hand term of \(\vec{j}, eD_e \nabla n_e\) is only calculated in the thermal perturbation layer. \(D_e\) the electron diffusion coefficient, is evaluated as \(\sigma k_b T_e / n_e\).

### 2.2.2. Ion/electron particle flux in the thermal perturbation layer.

In the arc column, electrons and the neutral atoms are evaluated under conditions of 2T Saha equilibrium. In the
In our 2D thermal perturbation layer, the quasi-neutrality state of the plasma is assumed, and primary ions/electrons and atoms are taken into account. Then, the electron and ion flux can be calculated approximately as: \( \Gamma_e = -D_{amb} \nabla n_e + n_e \bar{v} - \bar{E} \) and \( \Gamma_i = -D_{amb} \nabla n_i + n_i \bar{v} \). This leads to the solution of the ambipolar diffusion equation:

\[
\nabla \cdot (-D_{amb} \nabla n_e + n_e \bar{v}) = n_i, \tag{18}
\]

The net ionization rate is \( n_i = \alpha_{e} (G n_a - n_e^2) \), \( G \) is the Saha function \( G = n_{e,saha} / n_{a,saha} \), \( n_{e,saha} \) and \( n_{a,saha} \) are, respectively, the number densities of the electrons and the neutral atoms under conditions of 2T Saha equilibrium, the body recombination coefficient \( \alpha_{e} \) is taken from \([21]\), the ambipolar diffusion coefficient \( D_{amb} \) is calculated from the method of \([56]\), and \( \sigma \) in the thermal perturbation layer is expressed as: \( \sigma \approx \sigma_e = \frac{c_{pe}}{m_{e} n_{e}} \), where the electron collision frequency \( c_{pe} = n_{e} a_{e} + n_{i} a_{i} \).

### 2.2.3. Cathode body

Heat transfer equation is solved in the cathode body and reads:

\[
\nabla \cdot (-k_{ca} \nabla T_{ca}) = \bar{j} \cdot \bar{E}, \tag{19}
\]

where \( k_{ca} \) represent the thermal conductivity of cathode materials, and \( T_{ca} \) is the cathode temperature.

In the cathode body the electric potential and magnetic vector potential equations are same as those given in 2.2.1.

### 2.3. Calculation domain and boundary conditions

In this study, the free burning argon arc configuration is simplified as schematically shown in figure 2. A dc argon arc burns between a pure tungsten rod cathode and a flat anode. The diameter of the cathode is 12 mm, while the length of it is 10 mm, which corresponds to the literature \([38]\). The boundary conditions of the primary variables are summarized in table 2. At the axis of symmetry (A–B–C) of the domain, the axial symmetric conditions are employed. E–F is the wall, where a uniform temperature \( T_{in} = 300 \text{ K}, T_{cs} = 1000 \text{ K} \) is assumed. At A–F, the cathode bottom, the electric potential is set to be zero and the temperature is assumed to be uniform \( (300 \text{ K}) \) as previous work \([38]\) did. C–D is the anode bottom, where the temperature is set to be 1000 K and the electric potential is set to be zero. At the outlet D–E, a zero gradient of each primary variable is imposed with a backflow gas of a constant temperature \( (1000 \text{ K}) \). Though at outlet some simulations \([14, 57]\) introduce conditions of \( \frac{\partial T}{\partial m} = 0 \) in spite of backflow, the differences are small and limited in the near-outlet area.

At B–G–F, the plasma/cathode interface, there are in fact two interfaces. One is the interface between the cathode and sheath: the normal component of the electric current density is continuous; the heating flux to cathode is set as \( q_{net} \); the heavy-particle temperature equals the cathode surface temperature. The other one is the interface between the ionization layer and the thermal perturbation layer: for the electron energy equation, a heating flux of: \( q_e = k_e \nabla T_e \cdot n + (2.5 + 0.78) k_B T_e \frac{\gamma}{T_e} \) is applied (derived from equation (5)); following works \([26, 57]\), the heavy-particle temperatures and the normal component of the electric current density are continuous. Bohm’s criterion \( (\nu_B \) is the Bohm velocity, see equation (A.2) in appendix A) is adopted for the sheath/ionization layer boundary, which is explained in appendix A. The boundary condition \( \frac{\partial T}{\partial m} = 0 \) is used for the ion density calculation (2D) on the bulk plasma side, because we assume that the ion diffusion flux towards the cathode surface is majorly influenced by the ionization layer ion number density gradient (which is treated as 1D), and calculated using the equations from \([34]\).

### 3. Results and discussions

Governing equations are solved in FLUENT software using the SIMPLE algorithm. The transport properties of argon were calculated by Zhang \([58]\).

#### 3.1. Heating of the thermionic cathode

In the present work, two stable modes of solution (diffuse mode and spot mode) are obtained for arc currents 50–600 A. The two different types of cathode modes (diffuse and spot) were first obtained in \([59]\). Radial variation of the cathode voltage drop was also taken into account in this work and the following ones \([3, 46, 50–53]\). The two different modes are obtained from different initial conditions. In the calculation, the cathode arc attachment in the spot mode is limited in \( S < 6 \text{ mm} \) and when \( S > 6 \text{ mm} \) we introduce \( \frac{\partial T}{\partial m} = 0 \) as the boundary condition for the electron energy equation.

The cathode surface temperature/electron temperature along \( B–G–F \) is illustrated in figure 3 for \( I = 50 \text{ A}, 200 \text{ A}, 400 \text{ A} \). In figure 3, ‘S’ represents the distance from the center of the cathode head measured along the generatrix. The maximum cathode temperature appears in the cathode head fringe (G) for diffuse mode and in cathode head center (B) for the spot mode, where the total current density also reaches its maximum (figures 4 and 5). The maximum value of the cathode temperature reaches 3940 K, 3961 K, 3973 K for arc current 50 A, 200 A, 400 A in spot mode, and 3367 K, 3668 K, 3794 K in diffuse mode. The electron temperature in the diffuse mode is much lower than that in the spot mode. The maximum value of electron temperature reaches 30499 K, 31117 K, 33007 K for arc current 50 A, 200 A, 400 A in spot mode and 18934 K, 19608 K, 17944 K in diffuse mode. However, it has been reported in previous numerical work \([38, 39]\) that the spot mode of an arc discharge has two branches: there exists a solution of the spot mode in which both the center and fringe of the cathode head has a temperature peak. The present work could not obtain a stable second spot mode of the arc discharge; this is caused by a numerical problem, because, as
and in diffuse mode, and ..., in spot mode, respectively. The maximum of its ion heating component is located at the cathode surface B–G–F: diffuse mode, $I = 400$ A. The maximum of the total current density is equal to $1.53 \times 10^4$ A/m$^2$, and electron back diffusion current density $j_{bd}$ and electron thermal emission current density $j_{em}$ along the cathode surface B–G–F: diffuse mode, $I = 400$ A. 

Figure 3. Distributions of total current density $j$ and its components (ion current density $j_i$, electron back diffusion current density $j_{bd}$ and electron thermal emission current density $j_{em}$) along the cathode surface B–G–F, for different arc currents: $I = 50$ A, 200 A, 400 A.

Figure 4. Distributions of total current density $j$ and its components (ion current density $j_i$, electron back diffusion current density $j_{bd}$ and electron thermal emission current density $j_{em}$) along the cathode surface B–G–F: diffuse mode, $I = 400$ A.

indicated in paper [60], a discharge mode can join another mode or turn back, but not terminate suddenly.

Distributions of total current density $j$, and its components (ion current density $j_i$, electron back diffusion current density $j_{bd}$ and electron thermal emission current density $j_{em}$) along the cathode surface B–G–F, are plotted in figures 4 and 5 for $I = 400$ A. The maximum of the total current density is equal to $1.31 \times 10^4$ A/m$^2$ and $4.34 \times 10^3$ A/m$^2$ for the diffuse mode and spot mode, respectively. The maximum of the ion current density is equal to $4.85 \times 10^3$ A/m$^2$ and $9.87 \times 10^2$ A/m$^2$ for the diffuse mode and spot mode, respectively. As shown in figures 4 and 5, for the diffuse mode, the electron back diffusion current is about half of the ion current density. Thus we can approximately suppose $j_{tot} \approx j_{em} + j_i$ for the diffuse mode.

The total heating flux to cathode surface ($q_{tot}$) and its components (the densities of energy fluxes delivered to the cathode surface by ions $q_i$, by back-diffusion plasma electrons $q_{bd}$, by emission electrons $q_{em}$, and by radiation $q_{rad}$) along the cathode surface are shown in figures 6 and 7, for $I = 400$ A. The maximum of the total heating flux is equal to $9.64 \times 10^3$ W m$^{-2}$ and $2.07 \times 10^4$ W m$^{-2}$ for the diffuse mode and spot mode, respectively. The maximum of its ion heating component is equal to $1.53 \times 10^3$ W m$^{-2}$ and $3.04 \times 10^3$ W m$^{-2}$ for the diffuse mode and spot mode, respectively. For the diffuse mode, the maximum value of each heating flux is located at the fringe of the cathode head, where the temperature and current density also reach a maximum. However, for the spot mode the maximum value of the total heating flux is located at $S = 1.75$ mm, while the maximum temperature appears in the center of the cathode. As figures 6 and 7 show, the electron thermal emission has a much more obvious cooling effect on the cathode than thermal radiation; The plasma back-diffusion electron heating is nearly zero in the diffuse mode, but nevertheless has quite an important role in the spot mode of the arc discharge.

The calculations of $q_{bd}$, $q_{em}$, $q_i$ and $q_{rad}$ are the same as the decoupled method of Benilov [39], however in the present work thermal conduction from the plasma is taken into account and the contribution of the thermal conduction part is shown in section 3.2 for the convenience of discussion and reading.

3.2. Thermal influence from the bulk plasma

The typical ‘decoupled’ methods of cathode heating modeling [34, 37, 47] assume that the influence of the bulk plasma on the boundary layer can be neglected. However in this paper, the density of the net energy flux to the cathode surface generated in the near-cathode region is evaluated considering the influence of thermal conduction from bulk plasma. The densities of energy fluxes due to heat conduction of heavy particles ($q_{hc}$), electrons ($q_{ec}$) and the electron enthalpy transport ($q_{enth}$) are plotted in figures 8 and 9 for $I = 50$ A and figures 10 and 11 for $I = 400$ A. The expressions of each energy flux component are: $q_{hc} = -k_e \nabla T_e \cdot \vec{n}$; $q_{ec} = -k_e \nabla T_e \cdot \vec{n}$; $q_{enth} = (2.5 + 0.76) \frac{m_e c^2}{\rho_e} j_e$.

For $I = 50$ A, the maximum absolute value of $q_{tot}$, $q_{hc}$ and $q_{ec}$ reaches $2.08 \times 10^3$ W m$^{-2}$, $6.26 \times 10^3$ W m$^{-2}$ and $1.66 \times 10^3$ W m$^{-2}$ in diffuse mode, and $5.93 \times 10^3$ W m$^{-2}$, $9.61 \times 10^3$ W m$^{-2}$ and $1.02 \times 10^4$ W m$^{-2}$ in spot mode, respectively. For $I = 400$ A, the maximum absolute value of $q_{tot}$, $q_{hc}$ and $q_{ec}$ reaches $9.67 \times 10^3$ W m$^{-2}$, $8.80 \times 10^4$ W m$^{-2}$ and $1.31 \times 10^4$ W m$^{-2}$ in diffuse mode, and $2.07 \times 10^5$ W m$^{-2}$,
Therefore as figures 8–11 show, no matter whether in a low current (50 A) or a high current (400 A) situation, the energy fluxes due to heat conduction of heavy particles and electrons are not negligible ones. Nor is the sum of them. This conclusion is in agreement with simulations from [32]. The thermal conduction from the bulk plasma is as important as the electron enthalpy transport. At the same time the figures indicate that a substantial part of the electric power generated in the near-cathode layer is transported to the arc column (equals $q_{\text{enthalpy}} - q_{\text{tot}}$), which is also justified in decoupled numerical calculations [26].

### 3.3. Electrical influence from bulk plasma

As mentioned in the introduction, decoupled modeling of cathode heating also assumes that the voltage drop of the near-cathode layer ($U_{\text{tot}} = U_i + U_d$) is uniform in the whole arc root attachment [25]. This assumption stands on the condition that the voltage drop of the bulk plasma does not vary much around the arc root attachment. In figures 12–15 each voltage drop distribution is illustrated along the cathode surface for $I = 50$ A, 400 A: $U_d$ represents the sheath layer voltage drop; $U_i$ represents the ionization layer voltage drop; $U_i$ represents bulk plasma voltage drop, i.e. the absolute value of the plasma potential; $U_{\text{tot}}$ is the near-cathode layer voltage drop, which equals the sum of $U_i$ and $U_d$.

For the low current (50 A) situation, the variation of the bulk plasma voltage drop is very limited (~0.3 V) in the diffuse mode and a small value (~4 V) in the spot mode. Considering the fact that the voltage drop of the near-cathode layer $U_{\text{tot}}$ is as large as ~50 V, we can approximately suppose that $U_{\text{tot}}$ is uniform in the whole arc root attachment. However, for the high current (400 A) situation, the variation in the bulk plasma voltage drop reaches ~1.8 V in the diffuse mode and ~7 V in the spot mode. Therefore, it is physically inappropriate to
neglect the electrical influence from the bulk plasma for high current arc discharge in the spot mode.

The calculated voltage–current curves are plotted in figure 16 for $I = 50–600$ A, compared with the result from the decoupled method [38]. For a certain current, the near-cathode layer voltage drop is not uniform along the cathode surface in the present work. Thus the voltage drops given in figure 16 represent the minimum value of $U_{\text{tot}} = U_i + U_d$ (where $U_d$ reaches the maximum) for a certain total current.
10. U_{\text{tot}} is larger in the diffuse mode than in the spot mode for the same current in this work. However, in Benilov’s work, the result is just the opposite, except for a small range of high current. As figure 16 shows, the near-cathode layer voltage drop decreases with the increase of the total current. On the other hand the maximum bulk plasma voltage drop increases with the increase of the total current [25]. Therefore we can speculate that the uniform $U_{\text{tot}}$ assumption of the decoupled method should be more physically unrealistic with higher total current in the spot mode.

4. Conclusions

An atmospheric free-burning arc in argon is studied, using the code developed on the basis of a commercial CFD code FLUENT. In this paper the numerical model is based on a previous one [48], which calculates the argon arc simultaneously with the cathode body. The near-cathode layer modeling is coupled with the arc column modeling through setting a transitional layer in the 2D description between them. The arc gap between the cathode and anode is set to 5 mm. The total current is set to 50–600 A. Results are given for a pure tungsten rod cathode of 10 mm length and 6 mm radius.

(a) The thermal emission electrons cools the cathode surface, while the ion flux and back-diffusion plasma electrons heat it.

(b) At the cathode surface, the energy fluxes due to the heat conduction of heavy particles and electrons are comparable to the total heating flux, no matter whether the arc discharge is performed in a high (400 A) or low current (50 A) situation.

(c) Electrical influence from the bulk plasma on the cathode heating modeling becomes essential in the high current situation (>400 A) for the spot mode.

(d) The near-cathode layer voltage drop ($U_{\text{tot}}$) curves are similar to those of the decoupled method. However, $U_{\text{tot}}$ is larger in the diffuse mode than in the spot mode for the same current in this work, which is contrary to the results of decoupled modeling.

The results obtained are promising, which indicates that the coupled method of arc modeling differs significantly from the decoupled one. We plan now to work in the following direction: the use of the model in other cathode configurations and in 3D transient state assumption.

Acknowledgments

We thank Dr He-Ping Li (Department of Engineering Physics, Tsinghua University, E-mail: liheping@tsinghua.edu.cn) for careful advice. The discussion with him helped a lot in the writing of this paper.

The work is supported by The National Natural Science Foundation of China: NSFC 11475174, NSFC 11035005, NSFC 50876101.

Appendix A. Sheath and ionization layer

In this section a summary is given of equations describing the near-cathode layer of the plasma. The near-cathode plasma layer comprises two sub-layers: a space-charge sheath, which is adjacent to the cathode surface, and an ionization layer, which is adjacent to the sheath.
The space-charge sheath is considered as collisionless for ions. In the sheath, the ions moving to the cathode and electrons emitted from the cathode are accelerated. The emitted electron flux is found to have an upper limit [61, 62], and the initial energy of the positive ions at their entry plane is found to depend on the emitted electrons.

The ion current flux to the cathode is generated in the ionization layer [63], which is evaluated as:

\[ j_i = e n_{i,n} v_B \]  
(A.1)

where \( v_B \) represents the initial energy of the positive ions at their entry plane of the sheath:

\[ v_B^2 = \frac{k_B T_e}{m_i} \left( 1 - \mu \eta \right)^{-\frac{1}{2}} \left( 1 + \frac{1}{2} \eta^{-1} \right) \]  
(A.2)

\( n_{i,n} \) represents the ion density at the sheath edge:

\[ \frac{n_{i,n}}{n_{i,\infty}} = \frac{\alpha C_2 \sqrt{1 + \beta}}{C_2 + 2 \alpha C_2 \sqrt{1 + \beta} + \alpha^2 \sqrt{1 + \beta}} \]  
(A.3)

\( n_{i,\infty} \) is evaluated using the Saha equation for \( T_e, T_h \), and pressure \( p \) (1 atm in this paper) at the interface between ionization layer and thermal perturbation layer. The detailed definitions and calculations for the dimensionless coefficients \( C_2, \alpha \) and \( \beta \) are given in Benilov’s theory of the ionization layer [56, 63]. In addition, the detailed definitions and calculations for the dimensionless variables \( \mu, \eta, C_2 \) are given in the sheath theory for hot cathodes [61, 62].

The current flux of plasma electrons, which come to the cathode surface from the ionization layer after having overcome the retarding electric field in the space-charge sheath, is:

\[ j_{ed} = e n_{p,n} \left( \frac{k_B T_e}{2 \pi m_i} \right)^{1/2} \exp \left( - \frac{e U_d}{k_B T_e} \right) \]  
(A.4)

where \( n_{p,n} = n_{i,n} - \frac{q_e (m_e)^2}{2 \pi m_e} (2 e U_p + 2 k_B T_e)^{1/2} \), represents the density of plasma electrons at the sheath edge. \( U_d \) represents the sheath voltage drop.

The electron emission current flux from the cathode surface by means of the Richardson–Schottky formula reads:

\[ j_{em} = e A j_T^2 \exp \left( \frac{- \Delta A}{k_B T_e} \right) \]  
(A.5)

where \( \Delta A = \frac{e E_c}{2 k_B T_e} \) is the Schottky correction to \( A \), the work function for the thermionic emission of the electrode surface, \( E_c \), i.e. the electric field at the cathode surface, is obtained by solving Poisson’s equation in the sheath zone [61]:

\[ \frac{1}{2} \frac{\varepsilon_0}{n_{i,\infty} a e E_c} = 2 \eta_0 \left( 1 + \frac{\eta_0}{\eta} \right)^{1/2} - \left[ 1 - \mu \eta \left( \frac{\eta_0}{\eta} \right)^{1/2} (1 - e^{-\eta_0}) \right. \]  
\[ - \left. 2 \mu (\eta_0 + \tau)^{1/2} - \tau^{1/2} \right) \]  
(A.6)

Figure B1. A comparison of the predicted temperatures of the cathode surface with the experimental measurement of Haidar and Farmer [1]: thoriated tungsten cathode with a 60° conical tip, arc current \( I = 100 \, \text{A}, 200 \, \text{A}, 300 \, \text{A} \). \( z \) represents the axial distance from the cathode tip.

\( \varepsilon_0 \) is the dielectric constant, and \( \eta_0 \) is a dimensionless variable [61].

In the solution process, if the right-hand side of equation (6) is smaller than zero, \( j_{em} \) is evaluated as:

\[ j_{em} = e n_{i,n} \sqrt{ \frac{2 k_B T_e}{m_i} } \frac{2 \eta_0 \left( 1 + \frac{\eta_0}{\eta} \right)^{1/2} - 1}{2 \mu \left( (\eta_0 + \tau)^{1/2} - \tau^{1/2} \right) - \mu (\eta_0 + \tau)^{1/2} (1 - e^{-\eta_0})} \]  
(A.7)

The voltage drop in the ionization layer \( U_i \) and the sheath voltage drop \( U_d \) are evaluated as:

\[ U_i = \frac{k_B T_e}{e} \ln \frac{n_{i,\infty}}{n_{i,n}}, U_d = U_{tot} - U_i, \]  
(A.8)

Here \( U_{tot} \) is the given total voltage drop.

One can see that the near-cathode layer description in this paper mainly combines the ionization layer theory of Benilov [56, 63] and the hot cathode theory of Askari et al [61, 62]. Benilov also developed a double sheath model [34, 64] in which the electron emission does not influence the sheath electric field. In Benilov’ s theory, the Bohm velocity is evaluated as \( v_B = \sqrt{ \left( \frac{8 e_0 k_B T_e}{m_i} \right)^{1/2} } \), different from equation (2). This evaluation equals equation (2), when \( T_e/T_i \) is taken as \( +\infty \) and \( j_{em} \) is taken as zero.

Appendix B. Comparison with experimental results

B.1. Case 1

To apply and examine our coupling model (thermal perturbation (TP)-sheath model), a typical free burning arc configuration (thoriated tungsten cathode with a 60° conical tip, arc gap 5 mm) was chosen, which corresponds to the conditions of experiment [1] and is in agreement with previous studies.
In the present model, the arc plasmas are divided into five regions: the solid cathode body, sheath, pre-sheath, arc column and the solid anode body. The anode sheath is neglected and the anode body is coupled with the arc column by setting a heat flux to the anode surface [21].

Figure B1 shows the comparison of calculated temperatures at the cathode surface and the experimental measurements from the literature [1]. The electron temperature and heavy particle temperature of the arc column are plotted in figure B2, compared with experimental points of Haddad and Farmer [22]. It can be seen that the calculated temperatures for different arc currents (100 A, 200 A and 300 A) are all in good agreement with the experimental data. The comparison of calculated values and experimental data proved that our coupling model is practical.

B.2. Case 2

The numerical simulations performed using the 2T-sheath model (a 2T bulk plasma in ionization equilibrium, with near-cathode layer modeling including a space charge layer) under the conditions of the experiment in [5] were reported in [25]. In this work, the numerical simulations under the same conditions are carried out by means of the TP-sheath model. In addition, a comparison between the results of the two models is given in this section.

A sketch of the geometry is shown in figure B3. The cathode in the calculation domain is a cylinder made of tungsten with a hemispherical tip. The water-cooled anode is flat and made of copper. The bottom of the cathode and the outer part of the anode are considered to have an ambient temperature of 300 K. The arc chamber is closed. The current performed in this work is in the range 20–200 A.

The predicted arc column voltage \( U_{ac}^{(TP)} \), the near cathode layer voltage drop \( U_{cl}^{(TP)} \), the total arc voltage \( U_{arc}^{(TP)} \), \( U_{ac}^{(2T)} \), \( U_{cl}^{(2T)} \), \( U_{arc}^{(2T)} \), arc voltage measured by the experiment.

Figure B4. \( U_{ac}^{(TP)} \), \( U_{ac}^{(2T)} \): the arc column voltage predicted by, respectively, the TP-sheath model and the 2T-sheath model. \( U_{cl}^{(TP)} \), \( U_{cl}^{(2T)} \): the near cathode layer voltage drop predicted by the TP-sheath model and the 2T-sheath model. \( U_{arc}^{(TP)} \), \( U_{arc}^{(2T)} \): the total arc voltage predicted by the TP-sheath model and the 2T-sheath model. \( U_{ac}^{(exp)} \): arc voltage measured by the experiment.

A sketch of the geometry is shown in figure B3. The cathode in the calculation domain is a cylinder made of tungsten with a hemispherical tip. The water-cooled anode is flat and made of copper. The bottom of the cathode and the outer part of the anode are considered to have an ambient temperature of 300 K. The arc chamber is closed. The current performed in this work is in the range 20–200 A.

The predicted arc column voltage \( U_{ac} \), the near cathode layer voltage drop \( U_{cl} \) and the total arc voltage \( U_{arc} = U_{ac} + U_{cl} \) calculated by means of the TP-sheath model and the 2T-sheath model are shown in figure B4. Experimental data of the arc voltage taken from [5] are also shown in figure B4. There are two forms of arc attachment in the experiment [5]: diffuse and contracted. However, the values of the arc voltage in the two forms do not differ significantly. Note that the values shown in figure 6 belong to the diffuse form. We can see that the values of the arc voltage first decrease with increasing current and then increase slightly in both the TP-sheath and 2T-sheath models, which is due to the decrease in the near cathode layer voltage drop and the increase in the voltage of the arc column with increasing current. These numerical results of the two models are not very different from each other and from the
the difference is below 2V at both low current and high current. The near cathode layer voltage drop calculated by the TP-sheath model is very similar to the one calculated by the 2T-sheath model. The values of the arc column voltage predicted by the two models are more different, especially in the case of lower currents. However, the difference does not exceed 2 V, even at low current.

References

[15] Bai B 2012 The simulation of magnetically dispersed arc plasma coupled with electrode Thermal Science and Engineering (Hefei: University of Science and Technology of China)
[43] Coulombe S and Meunier J L 2000 Theoretical prediction of non-thermionic arc cathode erosion rate including both vaporization and melting of the surface Plasma Sources Sci. Technol. 9 239–47
[52] Nemchinsky V 2015 Heat transfer to a cathode of a rotating arc Plasma Sources Sci. Technol. 24 035013
[58] Zhang X N, Li H P, Murphy A B and Xia W D 2013 A numerical model of non-equilibrium thermal plasmas. I. Transport properties Phys. Plasmas 20 033508
[60] Benilov M S 2014 Multiple solutions in the theory of dc glow discharges and cathodic part of arc discharges. Application of these solutions to the modeling of cathode spots and patterns: a review Plasma Sources Sci. Technol. 23 054019
[61] Askari S and Minoo H 2008 Analytical expression for the electric potential in the plasma sheath near an arc-cathode Phys. Plasmas 15 043503